Vietnam Journal of Mechanics, VAST, Vol. 34, No. 3 (2012), pp. 211 – 214

Short Communication

DUAL APPROACH TO AVERAGED VALUES OF FUNCTIONS

Nguyen Dong Anh
Institute of Mechanics, VAST

Abstract. Averaged values play major roles in the study of dynamic processes. The use of those values allows transforming varying processes to some constant characteristics that are much easier to be investigated. In order to extend the use of averaged values one may apply the dual approach which suggests the consideration of two different aspects of a problem in question. This short communication proposes new global and local averaged values functions based on the dual conception.

Key words: Dual, averaged, global - local.

1. INTRODUCTION

Averaged values play major roles in the study of dynamic processes. The use of those values allows transforming varying processes to some constant characteristics that are much easier to be investigated. From the mathematical point of view the averaging approach brings the complicate differential problems more simple algebraic ones. In order to extend the use of averaged values one may apply the dual approach recently proposed and developed in [1-2]. One of significant advantages of the dual conception is its consideration of two different aspects of a problem in question allows the investigation to be more appropriate. The main idea of this short communication is to propose new global and local averaged values functions based on the dual conception.

2. DUAL CONCEPTION TO AVERAGED VALUES

Let first reconsider briefly the basic idea of averaged values. In mathematics an average is a measure of the "middle" or "typical" value of a data set. It is thus a measure of central tendency. The most common statistic is the arithmetic mean. The concept of average of a data set can be extended to functions. The global average value (GAV) of an integrable deterministic function $x(t)$ on a domain $D : (0, d)$ is a constant value defined by

$$< x(t) >= \frac{1}{d}\int_0^d x(t)dt. \tag{1}$$

In many cases when the function $x(t)$ is periodic with period 2π the value d is taken as 2π and it leads to the averaged value of $x(t)$ over one period. Using the dual approach to averaged values one may suggest a consideration respect two aspects, namely, to local and global levels. Thus, the following notations can be introduced.

Notation 1. The local averaged value (LAV) at level r of a deterministic integrable function $x(t)$ is defined by

$$< x(t) >_r = \frac{1}{r}\int\limits_0^r x(t)dt. \tag{2}$$

It is seen from (2) that in stead of the constant global averaged value (1) one has local averaged values (2) as a function depending on level r. When $r = d$, LAV (2) leads to GAV (1). Thus, GAV is a particular value of LAV.

Comment 1. Notation (2) can be called LAV order 1, a particular case of LAV order n, defined by

$$< ... << x(t) >_{r_1} >_{r_2} ... >_{r_n} = \frac{1}{r_n}\int\limits_0^{r_n} ... \frac{1}{r_2}\int\limits_0^{r_2} \frac{1}{r_1}\int_0^{r_1} x(t)dtdr_1dr_2...dr_{n-1}. \tag{3}$$

Now the consideration of second aspect yields the Notation 2.

Notation 2. The global-local averaged value (GLAV) of a deterministic integrable function $x(t)$ is defined by

$$<< x(t) >_r >= \frac{1}{d}\int\limits_0^d < x(t) >_r dr = \frac{1}{d}\int\limits_0^d \frac{1}{r}\int_0^r x(t)dtdr. \tag{4}$$

It can be seen that in general GLAV (4) differs from GAV (1) since GAV (1) is the averaged value of original values of $x(t)$ while GLAV is the averaged value of all local averaged values of $x(t)$. From the notation 2 it would be expected that GLAV (4) might express averaged characteristics that could not be obtained from GAV (1).

Comment 2. Similar to LAV order n one may define GAV order n as global averaged value of LAV order n.

In the case of stationary random functions $x(t)$ the following notations can be introduced.

Notation 3. The local mean value (LoMeV) at level r of a stationary random function $x(t)$ is defined by

$$< x(t) >_r = \int\limits_{-r}^r xp(x)dx. \tag{5}$$

where $p(x)$ is the stationary probabilistic density function of $x(t)$.

It is seen from (5) that the classical constant global mean value (GMeV) of $x(t)$ is a particular value of LoMeV when $r = \infty$. It also noted that the local mean square error criterion (LOMSEC) was introduced in [3] and developed in [4].

Notation 4. The global-local mean value (GLoMeV) of a stationary random function $x(t)$ is defined by

$$<< x(t) >_r>= \lim_{d\to\infty} \frac{1}{d} \int_0^d < x(t) >_r dr = \lim_{d\to\infty} \frac{1}{d} \int_0^d \int_{-r}^{r} xp(x)dxdr. \tag{6}$$

Same comments as above can be given to LoMeV and GLoMeV of a stationary random function.

2.1. Example 1

Let $x(t) = t^n$ defined in $[0, d]$. One gets

$$< t^n >_r= \frac{1}{r} \int_0^r t^n dt = \frac{r^n}{n+1}, \quad << t^n >_r>= \frac{1}{d} \int_0^d \frac{1}{r} \int_0^r t^n dtdr = \frac{d^n}{(n+1)^2} \tag{7}$$

while

$$< t^n >= \frac{1}{d} \int_0^d t^n dt = \frac{d^n}{n+1}. \tag{8}$$

It is obtained that $<< t^n >_r> \neq < t^n >$.

2.2. Example 2

Let $x(t) = \cos \omega t$ defined in $[0, \infty)$. One gets

$$< \cos \omega t >_r= \frac{1}{r} \int_0^r \cos \omega t dt = \frac{\sin \omega r}{\omega r}, \quad << \cos \omega t >_r>= \lim_{d\to\infty} \frac{1}{d} \int_0^d \frac{\sin \omega r}{\omega r} dr = 0, \tag{9}$$

while

$$< \cos \omega t >= \lim_{d\to\infty} \frac{1}{d} \int_0^d \cos \omega t dt = 0. \tag{10}$$

In this case one gets $<< \cos \omega t >_r>=< \cos \omega t >= 0$.

3. APPLICATION TO NONLINEAR EQUATIONS

For illustration of possible uses of the proposed global-local averaged values consider the following equation

$$e(u(t, x)) = 0 \tag{11}$$

where e is a given operator, $u(t, x)$ is unknown defined in the domain $D : t \in [0, d], x \in [0, L]$. Let $u(a, t, x)$ is an approximate solution of (11) which depends on a constant parameter a. The value of a can be determined from some conditions, for example, the mean square minimum criterion

$$< e^2(u(a, t, x)) >\rightarrow \min_a \tag{12}$$

where the averaging operator $<\cdot>$ is taken as follows

$$< \cdot >= \frac{1}{d}\int_0^d \frac{1}{L}\int_0^L (\cdot) dxdt \tag{13}$$

Using the dual conception the global averaged value (12) can first be replaced by local averaged values as follows

$$< e^2(a,t,x) >_{r,s} \equiv \frac{1}{r}\int_0^r \frac{1}{s}\int_0^s e^2(a,t,x) dxdt \to \min_a \tag{14}$$

The solution of (14) will be a function of r and $s, a = a(r,s)$. Thus, the expected value of a can be suggested as the global averaged value of $a(r,s)$, i.e.,

$$a = \frac{1}{d}\int_0^d \frac{1}{L}\int_0^L a(r,s) dsdr. \tag{15}$$

It is seen from (15) that it expresses the contributions of all local values of $a(r,s)$. Further investigations will be given in next full publications.

4. CONCLUSION

In this short communication the main idea of the dual conception is further extended to suggest new global and local averaged values for studying varying processes. These averaged values contain the global averaged value (GAV) as a particular case. It can be seen that GAV is obtained from original values of $x(t)$ while global-local averaged value (GLAV) is obtained from all local averaged values of $x(t)$. It would be expected that GLAV might express averaged characteristics that could not be obtained from GAV.

ACKNOWLEDGEMENTS

The research reported in this paper is supported by Vietnam National Foundation for Science and Technology Development under Project 107.04-2011.13 (09 - Mechanics).

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Received April 16, 2012